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# Neutrino Self-Energy and Index of Refraction in Strong Magnetic Field: A New Approach

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## Abstract

The Ritus'  $E_p$  eigenfunction method is extended to the case of spin-1 charged particles in a constant electromagnetic field and used to calculate the one-loop neutrino self-energy in the presence of a strong magnetic field. From the obtained self-energy, the neutrino dispersion relation and index of refraction in the magnetized vacuum are determined within the field range  $m_e^2 \ll eB \ll M_W^2$ . The propagation of neutrinos in the magnetized vacuum is anisotropic due to the dependence of the index of refraction on the angle between the directions of the neutrino momentum and the external field. Possible cosmological implications of the results are discussed.

## I. Introduction

The main goal of this paper is to investigate the effects of magnetic fields on neutrino propagation, a topic that has recently received increasing attention. We are particularly interested in strong field effects. Its possible application to astrophysics, where fields of the order of  $10^{13}$  G, and even larger<sup>1</sup>, can be expected in supernova collapse and neutron stars, makes this subject worth of detailed investigation. Just to mention one of the several astrophysical applications of magnetic field effects in neutrino physics, one may recall the suggestion that the modification of the neutrino dispersion relation in a magnetized charged medium<sup>2,3,4</sup> could serve to explain the high velocity of pulsars<sup>5</sup>.

Moreover, the presence of strong magnetic fields could have influenced the propagation of neutrinos in the early Universe and have an imprint in neutrino oscillations at those epochs<sup>6</sup>. The existence of primordial magnetic fields (of the order of  $10^{24}$  G at the electroweak scale) in the early Universe seems to be needed to explain the recent observations of large-scale magnetic fields in a number of galaxies, in galactic halos, and in clusters of galaxies<sup>7</sup>. These primordial magnetic fields could be generated through different mechanisms, as fluctuations during the inflationary universe<sup>8</sup>, at the GUT scale<sup>9</sup>, or during the electroweak phase transition<sup>10</sup>, among others.

Calculations of neutrino self-energies taking into account non-perturbative effects of magnetic fields have been carried out in several works<sup>4,11,12,13</sup>, using the Schwinger method<sup>14</sup>. In the present paper, the Ritus' technique, which was originally developed for the electron self-energy in QED in the presence of electromagnetic backgrounds<sup>15, 16</sup> is extended to the case of spin-1 charged particles. The Ritus' method is based on a Fourier-like transformation that diagonalizes in the momentum space the Green's functions of the charged particles in the presence of a constant magnetic field. This approach<sup>15,16</sup> is particularly convenient for the strong field case, where one can constraint the calculation to the contribution of the lower Landau level (LLL).

In this work we calculate the vacuum (zero temperature ( $T = 0$ ), zero density ( $\mu = 0$ )) contribution of the neutrino dispersion relation at strong magnetic field ( $m_e^2 \ll eB \ll M_W^2$ ,  $m_e$  is the electron mass and  $M_W$  is the W-boson mass). As discussed below, such a strong field can be expected to exist in the neutrino decoupling era. One of our main results is the existence of an anisotropic propagation of neutrinos in the strong magnetic field, even

in the absence of a medium ( $\mu = 0$ ). The anisotropy is due to the dependence of the index of refraction on the angle between the directions of the neutrino momentum and the external field. We also find that the terms explicitly depending on the mass of the charged lepton are negligible small (of order  $1/M_W^4$ ), while the leading term results of order  $1/M_W^2$ , thus rather significant.

The plan of the paper is as follows. In Section II, for the sake of understanding and completeness, we review the Ritus' method for the Green's function of spin-1/2 particles in the presence of a constant magnetic field. Then, we extend this method to the spin-1 charged particle case in the background of a constant magnetic field (corresponding to the crossed electromagnetic field ( $\mathbf{E} \cdot \mathbf{B} = 0$ ) case). In Section III, we use the results of Section II to calculate, in momentum space, the one-loop neutrino self-energy in the presence of a constant magnetic field. The neutrino dispersion relation and index of refraction are obtained in Section IV in the strong-field approximation ( $m_e^2 \ll eB \ll M_W^2$ ). In Section V, we make our final remarks and discuss a possible cosmological realization of the adopted strong-field approximation.

## II. Green's Functions at $B \neq 0$ in the Momentum Representation

The diagonalization, structure and properties of the Green's functions of the electron and photon in an intense magnetic field were considered exactly in external and radiative fields by Ritus in Refs. [15] and [16]. Ritus' formulation provides an alternative method to the Schwinger approach to address QFT problems on electromagnetic backgrounds. In Ritus' approach the transformation to momentum space of the spin-1/2 particle Green's function in the presence of a constant magnetic field is carried out using the  $E_p(x)$  functions<sup>15,16</sup>, corresponding to the eigenfunctions of the spin-1/2 charged particles in the electromagnetic background. The  $E_p(x)$  functions plays the role, in the presence of magnetic fields, of the usual Fourier  $e^{ipx}$  functions in the free case. This method is very convenient for strong-field calculations, where the LLL approximation is plausible and for finite temperature calculations.

In this section we will extend the Ritus' method to the case of spin-1 charged particles. This extension will allow us to obtain a diagonal in momentum space Green's function for the spin-1 charged particle in the presence of a constant magnetic field. Our results are important to investigate

the behavior of charged W-bosons in strong magnetic fields.

## A. Electron Green's function

For the sake of understanding, we summarize below the results obtained by Ritus<sup>15,16</sup> for the spin-1/2 case. The Green's function equation for the spin-1/2 particle in the presence of a constant electromagnetic field without radiative corrections is given by

$$(\gamma \cdot \Pi + m_e) S(x, y) = \delta^{(4)}(x - y) \quad (1)$$

where

$$\Pi_\mu = -i\partial_\mu - eA_\mu^{ext}, \quad \mu = 0, 1, 2, 3 \quad (2)$$

Taking into account that

$$[S(x, y), (\gamma \cdot \Pi)^2] = 0 \quad (3)$$

it follows that  $S(x, y)$  will be diagonal in the basis spanned by the eigenfunctions of  $(\gamma \cdot \Pi)^2$

$$(\gamma \cdot \Pi)^2 \Psi_p(x) = -p^2 \Psi_p(x) \quad (4)$$

Since  $[(\gamma \cdot \Pi)^2, \Sigma_3] = [(\gamma \cdot \Pi)^2, \gamma_5] = [\Sigma_3, \gamma_5] = 0$ , one can easily find the eigenfunctions  $\Psi_p$  in the chiral representation, where  $\Sigma_3 = i\gamma_1\gamma_2$  and  $\gamma_5$  are both diagonal and have eigenvalues  $\sigma = \pm 1$  and  $\chi = \pm 1$ , respectively. The eigenfunctions are given by

$$\Psi_p(x) = E_{p\sigma\chi}(x) \Theta_{\sigma\chi} \quad (5)$$

with  $\Theta_{\sigma\chi}$  the bispinors which are the eigenvectors of  $\Sigma_3$  and  $\gamma_5$ . We are considering the metric  $g_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ .

In the crossed field case ( $\mathbf{E} \cdot \mathbf{B} = 0$ ), one can always select the potential in the Landau gauge

$$A_\mu^{ext} = Bx_1\delta_{\mu 2} \quad (6)$$

which corresponds to a constant magnetic field of strength  $B$  directed along the  $z$  direction in the rest frame of the system. The fermion eigenfunctions are then given by the combination

$$E_p(x) = \sum_{\sigma} E_{p\sigma}(x) \Delta(\sigma), \quad (7)$$

where

$$\Delta(\sigma) = \text{diag}(\delta_{\sigma 1}, \delta_{\sigma -1}, \delta_{\sigma 1}, \delta_{\sigma -1}), \quad \sigma = \pm 1, \quad (8)$$

and the  $E_{p\sigma}$  functions are

$$E_{p\sigma}(x) = N(n) e^{i(p_0 x^0 + p_2 x^2 + p_3 x^3)} D_n(\rho) \quad (9)$$

In Eq. (9),  $N(n) = (4\pi |eB|)^{\frac{1}{4}} / \sqrt{n!}$  is a normalization factor and  $D_n(\rho)$  denotes the parabolic cylinder functions<sup>17</sup> with argument  $\rho = \sqrt{2|eB|}(x_1 - \frac{p_2}{eB})$  and positive integer index

$$n = n(k, \sigma) \equiv l + \frac{\sigma}{2} - \frac{1}{2} \quad n = 0, 1, 2, \dots, \quad (10)$$

The integer  $l$  in Eq. (10) labels the Landau levels. In a pure magnetic background the  $\chi$  dependence of the eigenfunctions  $E_{p\sigma\chi}$  drops away.

The  $E_p$  functions satisfy

$$\gamma \cdot \Pi E_p(x) = E_p(x) \gamma \cdot \bar{p} \quad (11)$$

where

$$\bar{p}_\mu = \left( p_0, 0, -\text{sgn}(eB) \sqrt{2|eB|} l, p_3 \right) \quad (12)$$

One can easily check that these functions are both orthonormal

$$\int d^4x \bar{E}_{p'}(x) E_p(x) = (2\pi)^4 \hat{\delta}^{(4)}(p - p') \equiv (2\pi)^4 \delta_{kk'} \delta(p_0 - p'_0) \delta(p_2 - p'_2) \delta(p_3 - p'_3) \quad (13)$$

and complete

$$\sum_k d^4p E_p(x) \bar{E}_p(y) = (2\pi)^4 \delta^{(4)}(x - y) \quad (14)$$

Here we have used the notation  $\bar{E}_p(x) \equiv \gamma^0 E_p^\dagger \gamma^0$  and  $\sum_k d^4p = \sum_k \int dp_0 dp_2 dp_3$ .

Using the functions  $E_p$  as a new basis, we obtain, thank to the properties (11) and (14), a representation of the fermion Green's function in the presence of a constant magnetic field which is diagonal in  $p$

$$\begin{aligned} S(p, p') &\equiv \int d^4x d^4y \bar{E}_p(x) S(x, y) E_{p'}(y) \\ &= (2\pi)^4 \hat{\delta}^{(4)}(p - p') \frac{1}{\gamma \cdot \vec{p} + m_e} \end{aligned} \quad (15)$$

The main idea of the Ritus' approach is, therefore, to use the eigenfunctions  $E_p(x)$ , which correspond to the asymptotic states of the particles in the presence of a constant external electromagnetic field, to perform a Fourier-like transformation that diagonalizes the Green's functions in the momentum space. The advantage of the representation (15) is that the Green's function is simply given in terms of the eigenvalues (12).

## B. W-Boson Green's function

Let us consider now the electroweak theory in the presence of a constant magnetic field corresponding to the potential (6). We choose the following gauge conditions

$$F_A = \partial^\mu A_\mu \quad (16)$$

$$F_z = \partial^\mu Z_\mu + \alpha_z M_z \phi_3 \quad (17)$$

$$F_W^+ = D^\mu W_\mu^+ + i\alpha_W M_W \phi \quad (18)$$

$$F_W^- = D^{*\mu} W_\mu^- - i\alpha_W M_W \phi^* \quad (19)$$

with

$$D_\mu = \partial_\mu - ieA_\mu^{ext} \quad (20)$$

In the above expressions the customary notation<sup>18</sup> for the electroweak fields is used.

With the gauge conditions (16)-(19) the Green's function equation for the W-boson takes the form

$$\left[ (\Pi^2 + M_W^2) \delta_\nu^\mu - 2ieF^\mu{}_\nu + \left( \frac{1}{\alpha_W} - 1 \right) \Pi^\mu \Pi_\nu \right] G_\mu{}^\nu(x, y) = \delta^{(4)}(x, y) \quad (21)$$

where  $\Pi_\mu$  is given in Eq. (2).

To solve Eq. (21) it is convenient to perform first a rotation in the Lorentz space using the transformation matrix

$$P^\mu{}_\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{pmatrix} \quad (22)$$

which satisfies  $P^{-1} = \tilde{P}^*$ , and the following relations

$$\Pi_\mu P^\mu{}_\alpha = \Pi_\alpha \quad (23)$$

$$(P^\alpha{}_\mu)^{-1} [iF^\mu{}_\nu] P^\nu{}_\beta = -B (S_3)^\alpha{}_\beta \quad (24)$$

In the above expressions the notation

$$\Pi_\alpha = (\Pi_0, \Pi_+, \Pi_-, \Pi_3) \quad (25)$$

$$\Pi_\pm = (\Pi_1 \pm i\Pi_2) / \sqrt{2} \quad (26)$$

was introduced.  $S_3$  represents the diagonal spin-one matrix

$$S_3 = \text{diag}(0, 1, -1, 0). \quad (27)$$

After doing the rotation (22), the Green's function equation (21) can be written as

$$\left[ (\Pi^2 + M_W^2) \delta_\beta^\alpha + 2eB (S_3)^\alpha{}_\beta + \left( \frac{1}{\alpha_W} - 1 \right) \Pi^\alpha \Pi_\beta \right] G_\alpha{}^\beta(x, y) = \delta^{(4)}(x, y) \quad (28)$$

Now we can use the Feynman gauge<sup>19</sup>,  $\alpha_w = 1$ , and follow an approach similar to the spin-1/2 case in order to find a diagonal in  $p$  solution of Eq. (28).

We start by solving the eigenvalue equation

$$\hat{D}^\alpha{}_\beta \Phi_k^\beta(x) = \bar{k}^2 \Phi_k^\alpha(x) \quad (29)$$

where

$$\hat{D}^\alpha{}_\beta = (\Pi^2 + 2eBS_3)^\alpha{}_\beta \quad (30)$$

Because  $[\hat{D}, S_3] = 0$ ,  $\Phi_k^\alpha(x)$  can be taken as a common eigenfunction to  $\hat{D}$  and  $S_3$ . The eigenvalue equation for  $S_3$  is then given by

$$(S_3)^\alpha{}_\beta \Phi_k^\beta(x) = \eta \Phi_k^\alpha(x), \quad \eta = 0, \pm 1 \quad (31)$$

where  $\eta$  denotes the different spin projections. From (31) we can write

$$\Phi_k^\alpha(x) = F_{k\eta}(x) \mathbb{E}_\eta^\alpha, \quad (32)$$

In Eq.(32)  $\mathbb{E}_\eta^\alpha$  represents the eigenfunctions of  $S_3$  corresponding to the eigenvalues  $\eta = 0, \pm 1$ , in the following way

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \text{ for } \eta = 0, \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \text{ for } \eta = 1, \text{ and } \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \text{ for } \eta = -1 \quad (33)$$

Note that there is a degeneracy for  $\eta = 0$ .

The eigenvalue problem (29) reduces now to find  $F_{k\eta}(x)$  from the differential equation

$$(\Pi^2 + 2eB\eta - \bar{k}^2) F_{k\eta}(x) = 0, \quad \eta = 0, \pm 1 \quad (34)$$

From the definitions (25)-(26) for the  $\Pi$  operator in the rotated system, and taking into account Eqs. (2) and (6), we can propose

$$F_{k\eta}(x) = \exp(-ik_0x_0 + ik_2x_2 + ik_3x_3) f_{k\eta}(x_1) \quad (35)$$



Then,  $f_{k\eta}(x_1)$  should satisfy

$$\left(\partial_\xi^2 - \frac{\xi^2}{4} + \varepsilon\right) f_{k\eta}(\xi) = 0 \quad (36)$$

where

$$\xi = \sqrt{2|eB|}(x_1 + k_2/eB) \quad (37)$$

and

$$\varepsilon = \frac{1}{2|eB|} \left( \bar{k}^2 + k_0^2 - k_3^2 - 2eB\eta \right) \quad (38)$$

Eq. (36) is the harmonic oscillator equation. Its physical solution requires

$$f_{k\eta}(\xi) \rightarrow 0 \text{ for } \xi \rightarrow \infty \quad (39)$$

$$\varepsilon = n + 1/2, \quad n = 0, 1, 2, \dots \quad (40)$$

From the condition (40) and the definition (38), one has

$$\bar{k}^2 = -k_0^2 + k_3^2 + 2(n + 1/2)eB + 2\eta eB \quad (41)$$

Considering the mass shell condition  $\bar{k}^2 = -M_W^2$  in Eq. (41), we can write

$$k_0^2 = (2n + 1)eB - g_s eB \cdot S + k_3^2 + M_W^2 \quad (42)$$

Eq. (42) is the well-known energy-momentum relation for higher-spin charged particles in interaction with a constant magnetic field<sup>20</sup>. Here  $g_s$  is the gyro-magnetic ratio of the particle with spin  $S$ . For W bosons:  $g_s = 2$ . In (42) we can also identify the so called<sup>21,22</sup> “zero-mode problem” at  $eB > M_W^2$ . As known, at those magnetic fields a vacuum instability appears giving rise to a W-condensation<sup>22</sup>. In our calculations we restrict the magnitude of the magnetic field to  $eB < M_W^2$ , thus, no tachyonic modes will be present.

From condition (41), and taking into account that  $\eta = 0, \pm 1$ , it follows that

$$n = m - \eta - 1, \quad m = 0, 1, 2, \dots \quad (43)$$

Then, from (41) and (43) we can write

$$\bar{k}^2 = -k_0^2 + k_3^2 + 2(m - 1/2)eB, \quad m = 0, 1, 2, \dots \quad (44)$$

where  $m$  are the Landau numbers of the energy spectrum of the W bosons in the presence of the magnetic field.

The solution of Eq. (36) satisfying the conditions (39) and (40) is

$$f_{k\eta}(\xi) = 2^{-n/2} \exp(-\xi^2/4) H_n(\xi/2) \quad (45)$$

where  $H_n$  are Hermite polynomials. Finally, substituting this solution into Eq. (35) we can write

$$F_{k\eta}(x) = N(n) e^{i(p_0 x^0 + p_2 x^2 + p_3 x^3)} D_n(\xi) \quad (46)$$

with  $N(n)$  a normalization factor and  $D_n(\xi)$  the parabolic cylinder functions.

The functions (32), together with the mass shell condition (42), respectively define the wave function and energy-momentum relation for spin-1 particles in the presence of a constant crossed electromagnetic field ( $\mathbf{E} \cdot \mathbf{B} = 0$ ). The study of the parallel-field case ( $\mathbf{E} \parallel \mathbf{B}$ ) can be found in Ref. [23].

Similarly to the spin-1/2 case (Eq. (7)), we can now form the transformation matrix to momentum space for the W-boson Green's function,

$$[F_k(x)]_\alpha{}^\beta = \sum_{\eta=0,\pm 1} F_{k\eta}(x) [\Omega^{(\eta)}]_\alpha{}^\beta \quad (47)$$

where the basis matrices of the Lorentz space  $\Omega^{(\eta)}$  are explicitly given by

$$\Omega^{(\eta)} = \text{diag}(\delta_{\eta,0}, \delta_{\eta,1}, \delta_{\eta,-1}, \delta_{\eta,0}), \quad \eta = 0, \pm 1 \quad (48)$$

Notice that the only difference between the  $E_p(x)$  functions (Eq. (7)) and the  $F_k(x)$  functions (Eq. (47)) is given through the basis of their matrix spaces. That is, for spin-1/2 the transformation matrices are expanded in the spinorial basis  $\Delta(\sigma)$ , while in the spin-1 case (Eq. (47)) they are expanded in the Lorentz basis  $\Omega^\eta$ .

It can be easily shown that the  $F_k(x)$  are orthogonal

$$\int d^4x F_k(x) F_{k'}^*(x) = (2\pi)^4 \hat{\delta}^{(4)}(k - k') \quad (49)$$

with normalization factor given by  $N^2 = \sqrt{4\pi|eB|}/n!$ ; and complete

$$\sum_n \int d^4k F_k(x) F_k^*(y) = (2\pi)^4 \delta^{(4)}(x-y) \quad (50)$$

Using the completeness property (50), one can prove that the Green's function

$$G_F(x, y)_\alpha{}^\beta = \sum_n \int \frac{d^4k}{(2\pi)^4} F_k(x) \frac{\delta_\alpha{}^\beta}{\vec{k}^2 + M_W^2} F_k^*(y) \quad (51)$$

is a solution of Eq. (28) in the Feynman gauge.

We can use the matrix  $P$  (22) to perform a similarity transformation of the Green's function (51) in order to represent it in the rectangular Lorentz space as

$$G_F(x, y)_\mu{}^\nu = \sum_n \int \frac{d^4k}{(2\pi)^4} \Gamma_{k\mu}^\alpha(x) \frac{\delta_\alpha{}^\beta}{\vec{k}^2 + M_W^2} \Gamma_{k\beta}^\dagger{}^\nu(y) \quad (52)$$

where

$$\begin{aligned} \Gamma_{k\mu}^\alpha(x) &= P^\alpha{}_\gamma F_k(x) P^{-1\gamma}{}_\mu \\ &= \frac{1}{2} \begin{pmatrix} 2\mathcal{H}_0(x) & 0 & 0 & 0 \\ 0 & \mathcal{H}_+(x) & -i\mathcal{H}_-(x) & 0 \\ 0 & i\mathcal{H}_-(x) & \mathcal{H}_+(x) & 0 \\ 0 & 0 & 0 & 2\mathcal{H}_0(x) \end{pmatrix} \end{aligned} \quad (53)$$

and the  $\mathcal{H}_0, \mathcal{H}_\pm$  functions are given by

$$\mathcal{H}_0(x) = \mathcal{F}_{m-1}(x), \quad \mathcal{H}_\pm(x) = \mathcal{F}_{m-2}(x) \pm \mathcal{F}_m(x) \quad (54)$$

In Eq. (54)  $\mathcal{F}_n(x)$  represents the functions  $F_{k\eta}(x)$  evaluated at the different spin projections  $\eta = 0, \pm 1$ . That is, using the relation (43) and evaluating  $\eta$  on each spin projection, we obtain the different values of  $n$  appearing as subindexes of  $\mathcal{F}_n(x)$  in terms of the Landau levels  $m$ .

From Eq. (52) and the orthogonality condition (49), the W-boson Green's function in a constant magnetic field can be written in the Lorentz rectangular frame as the following diagonal function of momenta

Figure 1: The order- $g^2$  neutrino bubble graph. Solid line corresponds to the charged lepton Green function in a constant magnetic field and wiggly line corresponds to the W-boson Green function in a constant magnetic field.

$$G_F(k, k')_{\mu}{}^{\nu} = (2\pi)^4 \widehat{\delta}^{(4)}(k - k') \frac{\delta_{\mu}{}^{\nu}}{\overline{k}^2 + M_W^2} \quad (55)$$

The eigenvalue  $\overline{k}^2$  is given in Eq. (44). We conclude this section by stressing that the  $\Gamma_k(x)$  matrices play for the spin-1 particle Green's function the same role as the  $E_p(x)$  matrices did for the spin-1/2 ones.

### III. Neutrino self-energy in the $E_p - \Gamma_k$ representation

It is known<sup>2,4,12,13</sup> that, to lowest order, the neutrino self-energy in a magnetic field is given by the bubble diagram arising from the  $e - W$  loop (See fig. 1).

This diagram has been calculated using a perturbative expansion in the magnetic field at  $T \neq 0$  and  $\mu \neq 0$  ( $\mu$  is the chemical potential of electrons) in Ref. [2]. Using the Schwinger proper-time method<sup>14</sup>, which involves the magnetic interaction non-perturbatively, the  $e - W$  bubble has been calculated at  $T \neq 0$  and  $\mu \neq 0$  in Refs. [13], and in vacuum (i.e.  $T = 0$  and  $\mu = 0$ ) in Refs. [11,12]. When calculating the thermal contribution to the  $e - W$  bubble with  $\mu \neq 0$  or  $\mu = 0$ <sup>2,13</sup>, the interaction between the magnetic field and the charged W-bosons in the Green's function spectrum has been often neglected under the assumption that  $eB \ll M_W^2$  (in this approximation the magnetic field does not appear in the poles of the W-boson Green's function). The W-boson Green's function in the above mentioned approximation, known in the literature as the “contact approximation,” takes the form

$$G_0^{\mu\nu}(x, y) \simeq \Phi(x, y) \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot (x-y)} \frac{g^{\mu\nu}}{M_W^2} \quad (56)$$

where

$$\Phi(x, y) = \exp \left( i \frac{e}{2} y_\mu F^{\mu\nu} x_\nu \right) \quad (57)$$

is the well known phase factor depending on the applied field<sup>24</sup>. That is, in the contact approximation the interaction of the magnetic field with the W-bosons is restricted, in the W-boson Green's function, to the phase factor (57). The contact approximation must be carefully handled in vacuum ( $T = 0$  and  $\mu = 0$ ), since it causes severe ultraviolet divergences. To avoid those subtleties one can instead consider the modification of the Green's function of the W-boson due to the external magnetic field, on an equal foot with the electron, and then, only at the end of the calculation take into account that the W-boson mass is the largest scale in the problem. As shown below, this more careful approach will prove to be convenient and useful for the calculation of the vacuum contribution to the neutrino self-energy in the presence of a magnetic field.

## A. General formulation

To calculate the neutrino self-energy in the one-loop approximation we start from

$$\Sigma(x, y) = \frac{ig^2}{2} R \gamma_\mu S(x, y) \gamma^\nu G_F(x, y)_\nu{}^\mu L \quad (58)$$

where  $L, R = \frac{1}{2}(1 \pm \gamma_5)$ ,  $G_F(x, y)_\nu{}^\mu$  is the W-boson Green's function in the Feynman gauge (52), and  $S(x, y)$  is the electron Green's function (15), that can be expressed in the configuration space as

$$S(x, y) = \oint_t \frac{d^4 q}{(2\pi)^4} E_q(x) \frac{1}{\gamma \cdot \vec{q} + m_e} \bar{E}_q(y) \quad (59)$$

Since the neutrino is an electrically neutral particle, the transformation to momentum space of its self-energy can be carried out by the usual Fourier transform

$$(2\pi)^4 \delta^{(4)}(p-p') \Sigma(p) = \int d^4x d^4y e^{-i(p \cdot x - p' \cdot y)} \Sigma(x, y) \quad (60)$$

Substituting with (58), (52) and (59) in (60) we obtain

$$(2\pi)^4 \delta^{(4)}(p-p') \Sigma(p) = \frac{ig^2}{2} \int d^4x d^4y e^{-i(p \cdot x - p' \cdot y)} \left\{ R \left[ \gamma_\mu \left( \not{\sum}_l \frac{d^4q}{(2\pi)^4} E_q(x) \frac{1}{\gamma \cdot \bar{q} + m_e} \bar{E}_q(y) \right) \right. \right. \\ \left. \left. \gamma^\nu \left( \not{\sum}_m \frac{d^4k}{(2\pi)^4} \frac{\Gamma_{k\mu}^\alpha(x) \Gamma_{k\alpha}^{\dagger\nu}(y)}{\bar{k}^2 + M_W^2} \right) \right] L \right\} \quad (61)$$

Taking into account that the spinor matrices (8) satisfy the following properties

$$\Delta(\pm)^\dagger = \Delta(\pm), \quad \Delta(\pm) \Delta(\pm) = \Delta(\pm), \quad \Delta(\pm) \Delta(\mp) = 0$$

$$\gamma^\mu \Delta(\pm) = \Delta(\pm) \gamma^\mu, \quad \gamma^\perp \Delta(\pm) = \Delta(\mp) \gamma^\perp,$$

$$L \Delta(\pm) = \Delta(\pm) L, \quad R \Delta(\pm) = \Delta(\pm) R \quad (62)$$

where the notation  $\gamma^\mu = (\gamma^0, \gamma^3)$  and  $\gamma^\perp = (\gamma^1, \gamma^2)$  was introduced, and using the definitions (53), (54), we obtain from (61)

$$(2\pi)^4 \delta^{(4)}(p-p') \Sigma(p) = \frac{-ig^2}{2} \int d^4x d^4y \not{\sum}_l \frac{d^4q}{(2\pi)^4} \not{\sum}_m \frac{d^4k}{(2\pi)^4} \frac{e^{-i(p \cdot x - p' \cdot y)}}{(\bar{q}^2 + m_e^2)(\bar{k}^2 + M_W^2)}$$

$$\left\{ 2\bar{q}_\perp \cdot \gamma^\perp \left[ I_{m-1,l}(x) I_{m-1,l-1}^*(y) \Delta(-) + I_{m-1,l-1}(x) I_{m-1,l}^*(y) \Delta(+) \right] \right.$$

$$\left. + \bar{q}_\parallel \cdot \gamma^\parallel \left[ \left( I_{m-2,l}(x) I_{m-2,l}^*(y) + I_{m,l}(x) I_{m,l}^*(y) \right) \Delta(-) \right. \right.$$

$$\left. + \left( I_{m-2,l-1}(x) I_{m-2,l-1}^*(y) + I_{m,l-1}(x) I_{m,l-1}^*(y) \right) \Delta(+) \right]$$

$$\left. + \bar{q}_\mu \epsilon^{1\mu 2\nu} \gamma_\nu \gamma^5 \left[ \left( I_{m-2,l}(x) I_{m-2,l}^*(y) - I_{m,l}(x) I_{m,l}^*(y) \right) \Delta(-) \right. \right.$$

$$+ (I_{m-2,l-1}(x)I_{m-2,l-1}^*(y) - I_{m,l-1}(x)I_{m,l-1}^*(y)) \Delta(+)] \} L \quad (63)$$

In Eq. (63) we used the compact notation

$$I_{a,b}(x) = \mathcal{F}_a(x) E_b(x) \quad (64)$$

with  $\mathcal{F}_a(x) = F_{k\eta}(x)$  and  $E_b(x) = E_{p\sigma}(x)$ . The subindexes  $a$  and  $b$  in (64) represent the number  $n$ , given in Eqs. (43) and (10) respectively. In (63) the subindexes  $a$  and  $b$  were already written in terms of the Landau levels for the W-bosons ( $m$ ) and electrons ( $l$ ) with the help of Eqs. (43) and (10).

Note that, differently from the approach used in previous works<sup>2,4,13</sup>, the interaction between the magnetic field and the W-bosons is kept in (63) in the poles of the self-energy operator through the effective momentum  $\bar{k}^2$ .

Expression (63) is the general formula for the one-loop neutrino self-energy in a constant magnetic field of arbitrary strength in the Ritus' approach.

## B. Strong field approximation

From now on, we assume that the magnetic field strength is in the range  $m_e^2 \ll eB \ll M_W^2$ . Since in this case the gap between the electron Landau levels is larger than the electron mass square ( $eB \gg m_e^2$ ), it is consistent to use the LLL approximation for the electron ( $l = 0$ ). On the other hand, it is obvious that such an approximation is not valid for the W-bosons, so for them we are bound to maintain the sum in all Landau levels. In this approximation we have

$$(2\pi)^4 \delta^{(4)}(p - p') \Sigma(p) = \frac{-ig^2}{2} \int d^4x d^4y \int d^4q \sum_m \int d^4k \frac{e^{-i(p \cdot x - p' \cdot y)}}{(\bar{q}^2 + m_e^2)(\bar{k}^2 + M_W^2)}$$

$$\{ \bar{q}_\mu \cdot \gamma^\mu (I_{m-2,0}(x)I_{m-2,0}^*(y) + I_{m,0}(x)I_{m,0}^*(y)) \Delta(-) L$$

$$+ \bar{q}_\mu \epsilon^{1\mu 2\nu} \gamma_\nu \gamma^5 (I_{m-2,0}(x)I_{m-2,0}^*(y) - I_{m,0}(x)I_{m,0}^*(y)) \Delta(-) L \} \quad (65)$$

To perform the integrals in  $x$  and  $y$  in (65) we should take into account the formulas<sup>16,25</sup>

$$\int d^4x e^{-ip \cdot x} I_{m'l'}(x) = \frac{(2\pi)^4}{\sqrt{l'!} \sqrt{m'!}} \delta^3(k+q-p) e^{-\hat{p}_\perp^2/2} e^{-ip_1 \frac{q_2 - k_2}{2eB}} e^{-i \text{sgn}(eB)[(l'-m')\varphi]} J_{m'l'}^*(\hat{p}_\perp) \quad (66)$$

$$\int d^4y e^{ip \cdot y} I_{m'l'}^*(y) = \frac{(2\pi)^4}{\sqrt{l!} \sqrt{m!}} \delta^3(k+q-p) e^{-\hat{p}_\perp^2/2} e^{ip_1 \frac{q_2 - k_2}{2eB}} e^{i \text{sgn}(eB)[(l'-m')\varphi]} J_{m'l'}(\hat{p}_\perp) \quad (67)$$

where

$$\hat{p}_\perp \equiv \sqrt{\hat{p}_1^2 + \hat{p}_2^2}, \quad \varphi \equiv \arctan(\hat{p}_2/\hat{p}_1), \quad \hat{p}_\mu \equiv \frac{p_\mu \sqrt{2|eB|}}{2eB} \quad (68)$$

$$J_{m'l'}(\hat{p}_\perp) = \sum_{j=0}^{\min(l', m)} \frac{m'! l'!}{j!(l'-j)!(m'-j)!} [\text{sgn}(eB) \hat{p}_\perp]^{m'+l'-2j} \quad (69)$$

Thanks to the factor  $e^{-\hat{p}_\perp^2/2}$ , the contributions from large values of  $\hat{p}_\perp$  are exponentially suppressed in the electron LLL approximation. Thus, it is consistent to keep only the smallest power of  $\hat{p}_\perp$  in  $J_{m'l'}(\hat{p}_\perp)$  so that

$$J_{m'l'}(\hat{p}_\perp) \simeq \delta_{m',0} \quad (70)$$

After using Eqs. (66), (67) and (70) in (65), and integrating in  $k$ , one obtains for the neutrino self-energy in the electron LLL approximation

$$\Sigma(p) = -ig^2 \pi |eB| \sum_m \int \frac{d^2 q_\parallel}{(4\pi)^2} \frac{1}{(q_\parallel^2 + m_e^2)(\overline{q_m} - p^2 + M_W^2)}$$

$$\left\{ \overline{q}_\parallel \gamma^\mu (\delta_{m,2} \delta_{m,2} + \delta_{m,0} \delta_{m,0}) \Delta(-) L + \overline{q}_\mu \epsilon^{1\mu 2\nu} \gamma_\nu \gamma^5 (\delta_{m,2} \delta_{m,2} - \delta_{m,0} \delta_{m,0}) \Delta(-) L \right\} \quad (71)$$

where

$$\overline{q_m} - p^2 = (q_\parallel - p_\parallel)^2 + 2(m - 1/2)eB \quad (72)$$

Note that in this approximation the sum in the W-boson Landau levels is effectively reduced to the contribution of the two levels  $m = 0, 2$ .



Let us perform now the integration in the parallel momenta  $q_{||}$ . With this aim, we can use the Feynman parametrization to represent, after Wick rotation to Euclidean space and some variable changes, the integral in (71) as

$$p_{||} \int_0^1 z dz \int d^2 q_{||} \frac{1}{(q_{||}^2 + \mathcal{M}^2)^2} \quad (73)$$

where

$$\mathcal{M}^2 = m_e^2 + [M_W^2 + 2(m - 1/2)eB - m_e^2] z + p_{||}^2 (1 - z) z \quad (74)$$

Then, performing the integrations in  $z$  and  $q_{||}$  and taking explicitly the sum in  $m$ , we arrive at

$$\Sigma(p) = \frac{g^2}{(4\pi)^2} [(\lambda - \lambda'_1) p_{||} \cdot \gamma^{\parallel} + \lambda'_2 p_{\mu} \epsilon^{1\mu 2\nu} \gamma_{\nu} \gamma^5] \Delta(-) L \quad (75)$$

where

$$\lambda = \frac{|eB|}{M_W^2}, \quad \lambda'_1 = \frac{m_e^2 |eB|}{M_W^4} \ln\left(\frac{M_W^2}{m_e^2}\right) + \frac{|eB| - m_e^2}{M_W^4} |eB|, \quad \lambda'_2 = -\frac{4|eB|^2}{M_W^4} \quad (76)$$

The expression (75) can be rewritten in a more convenient way using the following covariant form

$$\Sigma(p) = \left[ a_1 \not{p}_{||} + a_2 \not{p}_{\perp} + b \not{\hat{B}} + c \widehat{\not{B}} \right] L \quad (77)$$

In (77)  $u_{\mu}$  is the four-velocity of the center of mass of the magnetized system (background) and  $\widehat{B}_{\mu} = B_{\mu}/|B_{\mu}|$ , where  $B_{\mu}$  is the magnetic field in covariant notation  $B_{\mu} = \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} u^{\nu} F^{\rho\lambda}$ . Notice that when a magnetic field is present, to form the structure of  $\Sigma$  we have to consider, in addition to the usual tensors  $p_{\mu}$ ,  $g_{\mu\nu}$ , and  $\epsilon_{\mu\nu\rho\lambda}$ , the vectors  $B_{\mu}$  and  $u_{\mu}$ . The four-velocity vector  $u_{\mu}$  can be introduced in this case because the presence of a constant magnetic field picks up a special Lorentz frame: the rest frame (on which  $u_{\mu} = (1, 0, 0, 0)$ ) where the magnetic field is defined ( $u_{\mu} F^{\mu\nu} = 0$ ).

In a trivial vacuum,  $\Sigma$  would depend only on the four-momentum  $p_{\mu}$  showing no difference between longitudinal and transverse components ( $a_1 = a_2$ )

in agreement with the Lorentz invariance of the system. However, when a nontrivial background is present, the structure of  $\Sigma$  is enriched with new terms related to the symmetries broken by the background. In the present situation, since the magnetic field introduces a special Lorentz frame, the four-velocity  $u_\mu$  is needed to rewrite the structure in a covariant way, a situation similar to the finite temperature case<sup>26</sup>. Moreover, because of its special direction in the 3-dimensional space, the magnetic field breaks one more symmetry: the  $O(3)$  rotational symmetry. This new symmetry breaking is responsible for the appearance of the structure associated to the unit vector  $\hat{B}$ .

Notice that the separation between longitudinal and transverse momenta contributions in  $\Sigma$  (i.e.  $a_1 \neq a_2$  in (77)), an effect normally occurring in the presence of a constant magnetic field, has also a covariant representation in terms of the basic tensors of the problem,

$$\not{p}_\parallel = p^\nu W_{\nu\rho} W^{\mu\rho} \gamma_\mu \quad (78)$$

$$\not{p}_\perp = p^\nu \widetilde{W}_{\nu\rho} \widetilde{W}^{\mu\rho} \gamma_\mu \quad (79)$$

where

$$W_{\nu\rho} = (u^\alpha \hat{B}^\beta - u^\beta \hat{B}^\alpha) \quad (80)$$

$$\widetilde{W}_{\nu\rho} = \frac{1}{2} \epsilon_{\nu\rho\alpha\beta} W^{\alpha\beta} \quad (81)$$

The coefficients  $a_1$ ,  $a_2$ ,  $b$ , and  $c$  in (77) are Lorentz scalars that depend on the parameters of the theory and the used approximation. From (75)-(76) one can see that in the strong-field approximation here considered their leading contributions in powers of  $1/M_W^2$  are given by

$$a_1 = \frac{g^2}{(4\pi)^2} \lambda, \quad a_2 = 0, \quad b = a_1 \chi_p, \quad c = a_1 \omega_p \quad (82)$$

with

$$\omega_p = p \cdot u, \quad \chi_p = p \cdot \hat{B} \quad (83)$$

It is clear from the above equations that the longitudinal and transverse neutrino modes of propagation behave quite differently. This means that the strong magnetic field gives rise to an anisotropy in the neutrino propagation that is reflected in a neutrino self-energy mainly depending on the spatial momentum parallel to the applied field (77), (82). We point out that in the weak-field approximation a neutrino anisotropic propagation was found in Ref [11], while in Ref. [12] the splitting, characteristic in the presence of a magnetic field, between longitudinal and transverse momentum components, was absent. The last result was a consequence of the mass shell condition for vacuum  $\gamma \cdot p = 0$  that was imposed through out the calculation in [12]. Moreover, it should be notice that no linear term in  $B$  was found in [12], contrary to the behavior reported in [11], and to the one we found in  $\lambda$  and  $\lambda'$  from Eqs. (82)-(83).

For the case of neutrino propagation in a magnetized medium ( $\mu \neq 0$ ), a self-energy structure similar to (77) has been reported<sup>2</sup>. However, in that case, the coefficients  $b$  and  $c$  are proportional to the difference between the electron and positron densities which are functions of the electron chemical potential.

#### IV. Neutrino dispersion relation and index of refraction at strong magnetic field

Using the results (77), (82) in the dispersion equation for neutrinos propagating in the external magnetic field

$$\det [\gamma \cdot p - \Sigma(p)] = 0 \quad (84)$$

one obtains the following solution

$$\omega_p \simeq |\vec{p}| (\pm 1 + a_1 \sin^2 \alpha). \quad (85)$$

In Eq. (85),  $\alpha$  is the angle between the direction of the neutrino momentum and that of the applied magnetic field. Positive and negative signs correspond to neutrino and antineutrino energies respectively.

To obtain the neutrino index of refraction  $n$ , we substitute (85) into the formula

$$n \equiv \frac{|\vec{p}|}{\omega_p} \quad (86)$$

to find

$$n \simeq 1 - a_1 \sin^2 \alpha \quad (87)$$

From Eqs. (85) and (87) it is clear that neutrinos moving with different directions in the magnetized space will have different dispersion relation and consequently, different index of refraction. That is, although the neutrinos are electrically neutral, the magnetic field, through quantum corrections, can produce anisotropic neutrino propagation. The order of the asymmetry is  $g^2 \frac{|eB|}{M_W^2}$ .

An asymmetric neutrino propagation depending on the difference between the number densities of electrons and positrons was previously found in a charged medium ( $\mu \neq 0$ )<sup>2</sup>. There, the asymmetric term changes its sign when the neutrino reverses its motion. This property was suggested<sup>5</sup> to be the cause of the peculiarly high velocities observed in pulsars. In our case, however, the asymmetric term in the dispersion relation (85) does not change its sign by changing  $\alpha$  by  $-\alpha$ . On the other hand, it is clear from (87) that neutrinos moving along the external magnetic field ( $\alpha = 0$ ) have index of refraction similar to the one for the free case ( $n = 1$ ), while the index of refraction for neutrinos moving perpendicularly to the direction of the magnetic field ( $\alpha = \pi/2$ ), has a maximum departure from the free-case value.

## V. Final remarks and cosmological applications

In this paper we have found the vacuum contribution ( $T = 0$ ,  $\mu = 0$ ) of the neutrino self-energy in the strong-field regime ( $m_e^2 \ll eB \ll M_W^2$ ). The obtained self-energy depends only on the longitudinal neutrino momentum  $p_{\parallel}$ . This fact is responsible of the strongly anisotropic neutrino propagation discussed in Sec. IV. Nevertheless, contrary to what occurs in the case with  $\mu \neq 0$ <sup>2</sup>, the asymmetric term in the dispersion relation maintains the sign when the neutrino reverses its motion. From (76) we can see that in our approximation the terms in  $\Sigma$  depending on the charged lepton mass  $m_e$  are negligible small ( $1/M_W^4$ ).

Our results may find applications in the physics of neutrinos in the early Universe. First of all, notice that the existence of strong magnetic fields in the early Universe seems to be a very plausible idea<sup>27</sup>, since they may be required to explain the observed galactic magnetic fields,  $B \sim 2 \times 10^{-6} G$  on scales of the order of  $100 kpc$ <sup>7</sup>.

The strength of the primordial magnetic field in the neutrino decoupling era can be estimated from the following reasoning. Based on constraints derived from the successful nucleosynthesis prediction of primordial  ${}^4He$  abundance<sup>28</sup>, as well as on the neutrino mass and oscillation limits, an upper bound for the magnetic field produced in the early Universe prior to primordial nucleosynthesis<sup>29</sup> has been predicted. A formula for the upper bound at the QCD phase transition is<sup>29</sup>

$$B_{QCD} \lesssim \frac{10^{21} G}{\sum_i m_{\nu_i}/eV} \quad (88)$$

Taking into account the cosmological constraint on the sum of stable neutrino masses  $\sum_i m_{\nu_i} \lesssim 100 eV$ , the relation (88) implies that at  $T_{QCD} = 200 MeV$  the estimated upper limit for the primordial magnetic field is

$$B_{QCD} \lesssim 10^{19} G \quad (89)$$

On the other hand, taking into account the magnetic field effect of increasing  $n \leftrightarrow p$  reaction rates in primordial nucleosynthesis, the magnetic field upper limit at the end of nucleosynthesis ( $T = 10^9 K$ ) is<sup>30</sup>

$$B_{NS} \lesssim 10^{11} G \quad (90)$$

These values are in agreement with the equipartition principle that states that the magnetic energy can only be a small fraction of the Universe energy density. This argument leads to the relation  $B/T^2 \sim 2$ .

Therefore, it is reasonable to assume that between the QCD phase transition epoch and the end of nucleosynthesis a primordial magnetic field in the range

$$m_e^2 \leq eB \leq M_W^2 \quad (91)$$

could have been present.

On the other hand, the early Universe, unlike the dense stellar medium, is almost charge symmetric ( $\mu = 0$ ), since the particle-antiparticle asymmetry in the Universe is believed to be at the level of  $10^{-10} - 10^{-9}$ , while in stellar material is of order one. Thus, to investigate neutrino propagation in cosmology, we would have to consider, in addition to the vacuum contribution,  $\Sigma_\nu^{T=0}(m_e, M_W, eB)$ , of the neutrino self-energy, the  $\mu = 0$  thermal contribution,  $\Sigma_\nu^{T\neq 0}(m_e, M_W, eB, T)$ ,

$$\Sigma_\nu(m_e, M_W, eB, T) = \Sigma_\nu^{T=0}(m_e, M_W, eB) + \Sigma_\nu^{T\neq 0}(m_e, M_W, eB, T) \quad (92)$$

Notice that, when dealing with possible cosmological applications of our results, the vacuum neutrino self-energy calculated in Sec. III using the LLL approximation of the fermion Green function should be taken as a qualitative result. To understand this, we recall that a reasonable primordial magnetic field in the neutrino decoupling era would satisfy

$$m_e^2 \ll eB \sim 2T^2 \quad (93)$$

For such fields, the effective gap between the Landau levels is  $eB/T^2 \sim \mathcal{O}(1)$ . Clearly, in this case the weak-field approximation, which would correspond to a high-temperature approximation, cannot be used because field and temperature are comparable. On the other hand, because the thermal energy is of the same order of the energy gap between Landau levels, it is barely enough to induce the occupation of only a few of the lower Landau levels. Therefore, the LLL approximation, even though too radical here since, strictly speaking, this is not a clear-cut strong-field case, it will provide a qualitative description of the neutrino propagation. In other words, the vacuum structure associated to the anisotropic propagation should still be present for fields  $eB \sim 2T^2$ , although the coefficient  $c$  in Eq. (77) may be quantitatively different.

A more quantitative treatment of the neutrino propagation in a field  $eB \sim 2T^2$  would require numerical calculations due to the lack of a leading parameter. In this sense, the extension to spin-one charged particles of the Ritus' method developed in Section II can be very useful, since expression (63) is a suitable representation to be used to numerically find the coefficients  $a$ ,  $b$  and  $c$  of the general structure of the self-energy (77).

Therefore, one can expect that the anisotropic propagation of neutrinos at zero chemical potential, found in our calculations at the LLL approximation, would be reflected in the propagation of neutrinos in the early universe if primordial fields satisfying the condition (93) were present. Even if the anisotropic term results small, it would account for a qualitatively new effect. If that is the case, one can envision that the anisotropy would leave a footprint in a yet to be observed relic neutrino cosmic background. If such an effect were detected, it would provide a direct experimental proof of the existence of strong magnetic fields in the early Universe.

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